Shortcuts to adiabaticity in quantum thermodynamics

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Unitary dynamics

Open quantum dynamics
Maximum efficiency of an Otto cycle

Quantum efficiency

\[ \mathcal{E} = -\frac{\langle W \rangle_1 + \langle W \rangle_3}{\langle Q \rangle_4} \]

Work probability distribution [Talkner, Lutz, Hanggi, PRE 75, 050102(R) (2007)]

\[ P(W; t) = \sum_{k,n} \delta[W - (\varepsilon_k(t) - \varepsilon_n(0))] p^t_{nk} p^n_0 \]
Maximum efficiency of an Otto cycle

Quantum efficiency

$$\mathcal{E} = - \frac{\langle W \rangle_1 + \langle W \rangle_3}{\langle Q \rangle_4}$$

Work probability distribution [Talkner, Lutz, Hanggi, PRE 75, 050102(R) (2007)]

$$P(W; t) = \sum_{k,n} \delta[W - (\varepsilon_k(t) - \varepsilon_n(0))] p^t_{nk} p^0_n$$

Maximum efficiency of an Otto cycle [Abah et al. PRL 109, 203006 (2012)]

$$\mathcal{E} = 1 - \frac{\omega_1 \coth(\beta_1 \hbar \omega_1 / 2) - Q^*_2 \coth(\beta_2 \hbar \omega_2 / 2)}{\omega_2 Q^*_1 \coth(\beta_1 \hbar \omega_1 / 2) - \coth(\beta_2 \hbar \omega_2 / 2)}, \quad \mathcal{E}_{\text{max}} = 1 - \frac{\omega_1}{\omega_2}$$

Attainable in the adiabatic limit
Vanishing output power! $Q^*_{1,2} \rightarrow 1$
Finite-time thermodynamics

The play dramatizes the trade-off between the output-power of a QHE and its efficiency.

\[ P = -\frac{\langle W \rangle_1 + \langle W \rangle_3}{\sum_{j=1}^{4} \tau_j} \]

and its efficiency

\[ \mathcal{E} = -\frac{\langle W \rangle_1 + \langle W \rangle_3}{\langle Q \rangle_4} \]
Shortcuts as a way out of the tragedy

Shortcuts to adiabaticity

Shortcuts to adiabaticity: Why speeding things up?

Quantum thermodynamics
heat engines
ground state cooling

Defect suppression in condensed matter systems and quantum simulation

Quantum Information
Quantum annealing
Quantum Optics
Control of decoherence, noise and perturbations
Shortcut to adiabaticity: Why speeding things up?

**Shortcut to adiabaticity:**

Fast non-adiabatic process to prepare a state mimicking adiab. dynamics


Processes: Expansion, transport, splitting, adiabatic passage, phase transitions, …

Systems: ultracold atoms, ions chains, quantum dots, spin systems, NVC, …

Experiments: Nice, NIST, Mainz, PTB, MPQ, Florence, Trento, Tsukuba, …
Shortcuts to adiabaticity

Ground state loading
In an optical lattice
Masuda, Nakamura, AdC PRL 113, 063003 (2014)

Ion transport
Deffner, Jarzynski, AdC PRX 4, 021013 (2014)

Adiabatic crossing of quantum phase transition
AdC, Rams, Zurek PRL 109, 115703 (2012)
Saberi, Opatrný, Mølmer, AdC, PRA 90, 060301(R)
AdC & Sengupta, EPJ ST 224, 189 (2015)

Topological Defect suppression
AdC et al. PRL105, 075701 (2010)
AdC, Kibble, Zurek, JPCM 25, 404210 (2013)

Ultracold Atom microscopy
AdC, EPL 96, 60005 (2011)
AdC, PRA 84, 031606(R) (2011)
AdC, PRL 111, 100502 (2013)

Quantum thermodynamics
Chen et al, PRL 104, 063002 (2010)

And many other applications
(chemical rate processes, quantum logic gates, soliton dynamics, atom interferometry, …)
Otto cycle: Shortcuts to adiabatic expansions?
Opening the trap

\[ \omega(t) = \omega_i \left[ 1 + \frac{\omega_f - \omega_i}{\omega_i} \tanh \frac{t}{\tau} \right] \]

from sudden to adiabatic

Excitation of the breathing mode of the cloud
1. Consider a time-dependent Hamiltonian harmonic oscillator

\[ \hat{H} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + \frac{1}{2} m \omega(t)^2 x^2 \]

\[ \hat{H} \phi_n(x) = E_n \phi_n(x) \]

2. Impose a self-similar dynamical ansatz

\[ \phi(x, t) = \frac{1}{b(t)^{1/2}} \exp \left[ \frac{i m \dot{b}(t)}{2 \hbar b(t)} x^2 - i \int_0^t \frac{E_n}{b(s)^2} ds \right] \phi \left[ \frac{x}{b(t)}, t = 0 \right] \]

3. Get the consistency equation: scaling factor as function of trap frequency

\[ \ddot{b} + \omega^2(t) b = \omega_0^2 / b^3 \]

1. Take a somewhat general many-body time-dependent Hamiltonian

\[ \hat{H} = \sum_{i=1}^{N} \left[ -\frac{\hbar^2}{2m} \Delta_i^{(D)} + \frac{1}{2} m \omega^2(t) x_i^2 \right] + \epsilon \sum_{i<j} V(x_{ij}) \quad x_i \in \mathbb{R}^D, \; x_{ij} = x_i - x_j \]

With a potential satisfying

\[ V(\lambda x) = \lambda^\alpha V(x) \]

2. Impose a self-similar dynamical ansatz

\[ \Phi(\{x_i\}, t) = \frac{1}{b^{D/2}} e^{i \sum_{i=1}^{N} \frac{m x_i^2 b}{2 \hbar} - i \mu \tau(t)/\hbar} \Phi(\{\frac{x_i}{b}\}, 0) \]

3. Get the consistency equations, i.e.

\[ \ddot{b} + \omega^2(t)b = \frac{\omega_0^2}{b^3} \quad \epsilon(t) = b^{\alpha-2} \]

AdC, PRA 84, 031606(R) (2011); PRL 111, 100502 (2013)
1. Reduce the scaling ansatz to the desired initial and final states

Boundary conditions:

\[ b(0) = 1, \quad \dot{b}(0) = 0, \quad \ddot{b}(0) = 0 \]

\[ b(\tau) = \sqrt{\frac{\omega_f}{\omega_0}}, \quad \dot{b}(\tau) = 0, \quad \ddot{b}(\tau) = 0 \]

2. Determine an ansatz for the scaling factor (e.g. a polynomial)

3. Find the driving frequency and coupling strength from the consistency equations

\[ \ddot{b} + \omega^2(t)b = \frac{\omega_0^2}{b^3} \]

\[ \epsilon(t) = b^{\alpha-2} \]

AdC, PRA 84, 031606(R) (2011); PRL 111, 100502 (2013)
Experiments: expansion of a thermal cloud & BEC

Suppression of breathing mode excitation

Protocol: shortcuts to adiabaticity

Linear vs shortcut BEC decompression

Labeyrie’s group @ Nice

Theory (single-particle)

Experiments (single-particle / mean-field BEC)
J.-F. Schaff et al. EPL 93, 23001 (2011)
Experiment: many-body shortcuts

Scaling of phonons and shortcuts to adiabaticity in a one-dimensional quantum system

W. Rohringer, D. Fischer, F. Steiner, I. E. Mazets, J. Schmiedmayer, and M. Trupka
1 Vienna Center for Quantum Science and Technology, Atominstitut, TU Wien, 1020 Vienna, Austria
2 Ioffe Physical-Technical Institute of the Russian Academy of Sciences, 194021 St. Petersburg, Russia
(Dated: December 23, 2013)

Shortcut vs standard expansion

Theory (quantum fluids)
AdC PRA 84, 031606(R) (2011); PRL 111, 100502 (2013)
Experiments (1D Bose gas)
Rohringer et al. arXiv:1312.5948
Example: superadiabatic quantum piston
Example: superadiabatic quantum piston

UT Austin all optical box
at Raizen’s Lab
PRA, 71, 041604(R) (2005)

\[ U_{\text{aux}} = -\frac{1}{2} m \ddot{b} x^2 \]

Quantum Piston

normal expansion

shortcut to adiabaticity

Jarzynski, PRA 88, 040101 (2013); AdC, PRL 111, 100502 (2013)
Quantum Heat Engine: Otto cycle

Nanodevice converting thermal energy into work

Example: lifting an ion in a trap

Unitary dynamics

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Superadiabatic quantum engine

STA in steps 1 & 2
Thermalization time $\ll$ adiabats

Thermodynamic cycle at finite power and zero friction
i.e., maximum efficiency

$$\mathcal{E}_{\text{max}} = 1 - \frac{\omega(\tau)}{\omega(0)}$$

Thermal state at $t=0$ in stroke 1)

$$\delta W = \langle W \rangle - \langle W_{\text{ad}} \rangle = \frac{1}{\beta_t} S(\rho_t \| \rho_{t}^{\text{ad}}), \quad \rho_{t}^{\text{ad}} = \sum_n p_n^0 |n(t)\rangle \langle n(t)|$$

![Diagram showing the Otto cycle](image)
Energy cost: time average excitation energy

\[ \delta W = \langle W \rangle - \langle W_{ad} \rangle \quad \langle \delta W \rangle = \frac{1}{\tau} \int_0^\tau \delta W \, dt \]

\( \omega_0 \tau \)

\( <\delta W> \sim 1/\tau \)

- Blue line: \( \beta = 1, \gamma = 2 \)
- Red line: \( \beta = 1, \gamma = 4 \)
- Green line: \( \beta = 5, \gamma = 2 \)
Performance: Ultimate Quantum Limits

How fast can we go?

Not faster than the Quantum Speed Limit

The speed at which a quantum state evolves is linked to the dynamics of the Hamiltonian

\[ E = \langle \Psi | H | \Psi \rangle \quad \Delta E = \sqrt{\langle \Psi | (H - E)^2 | \Psi \rangle} \]

Minimum time \( T \) required for a quantum state to evolve to an orthogonal state

\[ T \geq \frac{\pi}{2} \max \left( \frac{\hbar}{E}, \frac{\hbar}{\Delta E} \right) \]
Performance: Ultimate Quantum Limits

How fast can we go?

Not faster than the Quantum Speed Limit

\[ T \geq \frac{\pi}{2} \max \left( \frac{\hbar}{E}, \frac{\hbar}{\Delta E} \right) \]

Fundamental bound to the output power of a QHE

\[ \mathcal{P} \leq - \frac{\langle W_{ad,1}(\tau) \rangle + \langle W_{ad,3}(\tau) \rangle}{\hbar \mathcal{L} \left( \rho^\text{eq}_\tau, \rho_0 \right)} \max \left\{ E_\tau, \Delta E_\tau \right\} \]

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Unitary dynamics

Open quantum dynamics
**Time-energy uncertainty relation**

*Isolated* systems: unitary dynamics

\[ T \geq \frac{\pi}{2} \max \left( \frac{\hbar}{E}, \frac{\hbar}{\Delta E} \right) \]

*Real* systems: coupled to an environment

Nonunitary dynamics (master equation)

\[ \frac{d\rho_t}{dt} = \mathcal{L} \rho_t \]

What replaces energy in an open system?

Bound to the speed of evolution for open (as well as unitary) system dynamics

includes coupling to an environment

\[ E, \Delta E \rightarrow \sqrt{\text{Tr}[\mathcal{L}^\dagger \rho_0^2]} \]

AdC et al. PRL 110, 050403 (2013)
See too: Taddei et al. PRL 110, 050402 (2013)
Deffner et Lutz, PRL 111, 010402 (2013)
Summary

Shortcuts to adiabaticity speed up processes by tailoring excitations

Thermodynamic cycle working at finite power and zero friction

Quantum speed limits impose ultimate performance bounds
Workshop Details

Thermodynamics and Nonlinear Dynamics in the Information age

07/13/2015 - 07/17/2015

Sebastian Deffner - Los Alamos National Laboratory
Korana Burke - University of California Davis
Adolfo del Campo - University of Massachusetts Boston

MEETING DESCRIPTION:
Since its beginnings one of the main purposes of thermodynamics has been the optimization of devices. Commonly, processes are characterized as optimal if they are maximally fast or maximally efficient. Recent years have seen the development of various theoretical tools which tremendously broadened our understanding of such optimal processes, in quantum mechanics and in classical physics. A particular highlight are so-called shortcuts to adiabaticity -- finite time processes that mimic adiabatic dynamics without the requirement of slow driving. These exciting new results found relevance and application in a wide variety of fields including Quantum Sensing and Metrology, Finite-Time Thermodynamics, Quantum Simulation, Quantum Computation, Quantum Communication, and Quantum Optimal Control Theory. A second pillar of modern thermodynamic optimization are so-called information engines. In these systems the effects of information gain and its feedback into the dynamics are explicitly studied. As a consequence Maxwell demon-like systems have lost its demonic obscurity and have become an integral part of realistic optimization. All of these processes are frequently governed by inherently nonlinear equations. This conference aims at an exchange of ideas from researchers in Non-Equilibrium Thermodynamics, Atomic, Molecular, and Optical Sciences, Quantum Information and Quantum Technologies, Statistical Mechanics, Optimal Control Theory, and Nonlinear Dynamics.
Collaborators

Quantum Heat Engines
John Goold (ICTP)
Mauro Paternostro (Belfast)

STA in quantum fluids
Malcolm Boshier (LANL)
Sebastian Deffner (LANL)
Chris Jarzynski (UMD)

Quantum Speed limits
Inigo L. Egusquiza (Bilbao)
Martin B. Plenio (Ulm)
Susana F. Huelga (Ulm)

Opening: PhD & Postdoctoral Position

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